Limits and Continuity - AP Calculus

Solve each of the following:

$$f(x) = \begin{cases} x^2 - 3x + 9 & , x \leq 2 \\ kx + 1 & , x > 2 \end{cases}$$

1. The function f(x) is defined above. For which of the following values of k, if any, is f(x) continuous at $x = 2$?

   A) 1
   B) 2
   C) 3
   D) 4
   E) There is no value of k

$$f(x) = \begin{cases} x^2 - 3x \ , & x \geq 3 \\ \dfrac{3}{x-3} \ , & x < 3 \end{cases}$$

2. The function f(x) is defined above. Which of the following statements is true?

A) $\lim\limits_{x \to 3^+} f(x)$ exists

B) $\lim\limits_{x \to 3^-} f(x)$ exists

C) f is continuous at $x = 3$

D) f is differentiable at $x = 3$

$$f(x) = \begin{cases} 2x^2 - 5x + 3 & if \quad x \le 1 \\ \dfrac{1}{x-1} + 4 & if \quad x > 1 \end{cases}$$

3. The function f(x) is defined above. Determine if f(x) is continuous at $x = 1$.

A) Yes

B) No, there is a jump discontinuity

C) No, there is an infinite discontinuity

D) No, there is a removable discontinuity

4. Let g be the function defined by $g(x) = \frac{(x-3)(x^2-k^2)}{(k-x)(x^2-9)}$, where k is constant. For what value of k, if any, does $\lim_{x \to 3} g(x) = 2$

A) -3

B) 6

C) -15

D) There is no value of k

5. The function f is defined by $f(x) = \frac{|x| \cdot (sin(x)cos(x))}{x^3 - 8}$. At how many values of x does f have a discontinuity?

A) Zero

B) One

C) Two

D) Three

6. If f is a function such that $\lim_{x \to \infty} h(x) = 0$, which of the following could be $h(x)$?

A) $sin(x)$

B) $tan(x) \bullet cos(x)$

C) $\frac{x}{ln(x)}$

D) $\frac{5x^2+4x-3}{3x^3+2x^2-4}$

7. The function h is continuous at $x = 1$.

$$h(1) = \lim_{x \to 1^-} h(x) + \lim_{x \to 1^+} h(x) + 1.$$

Which of the following must be true?

A) $h(x)$ is differentiable at $x = 1$

B) $h(1) = -1$

C) $h(1) = 1$

D) $h(1) \neq \lim_{x \to 1} h(x)$

8. $\lim_{x \to -5} \frac{x+5}{|x+5|}$ is

A) -1

B) 1

C) 0

D) nonexistent

9. Let f be the function given by $f(x) = \frac{x-1}{5|x-1|}$. Which of the following is true?

A) $\lim\limits_{x \to 1} f(x) = \frac{1}{5}$

B) $f(x)$ has an infinite discontinuity at $x = 1$

C) $f(x)$ has a removable discontinuity at $x = 1$

D) $f(x)$ has a jump discontinuity at $x = 1$

10. $\lim_{x \to \infty} \frac{\sqrt{16x^4+3x-2}}{2x^2-5x+1}$ is

A) 2

B) 0

C) 8

D) infinite