Derivatives - AP Calculus

Solve each of the following:

1. If $y = \sqrt[3]{(x^2 + 1)^2}$, then $\frac{dy}{dx} =$

A) $\frac{4x}{(x^2+1)^{\frac{1}{3}}}$

B) $\frac{4x}{(x^2+1)^{-\frac{1}{3}}}$

C) $\frac{4x}{3(x^2+1)^{\frac{1}{3}}}$

D) $\frac{2}{3(x^2+1)^{\frac{1}{3}}}$

2. $\lim_{h \to 0} \frac{tan(\frac{\pi}{3}+h)-tan(\frac{\pi}{3})}{h} =$

A) 1

B) 2

C) $\frac{\sqrt{3}}{2}$

D) 4

$$f(x) = \sqrt{\tan^{-1}\left(\left(3x^2 - x\right)^2\right)}$$

3. The function f(x) is defined above. Find $f'(x)$

A) $\frac{1}{2}(tan^{-1}((3x^2 - x)^2))^{\frac{-1}{2}} \bullet \frac{1}{1+(3x^2-x)^4} \bullet 2(3x^2 - x)(6x - 1)$

B) $(tan^{-1}((3x^2 - x)^2))^{\frac{-1}{2}} \bullet \frac{1}{1+((3x^2-x)^2)^2} \bullet 2(3x^2 - x)(6x - 1)$

C) $\frac{1}{2}(tan^{-1}((3x^2 - x)^2))^{\frac{-1}{2}} \bullet \frac{1}{1+(3x^2-x)^2} \bullet 2(3x^2 - x)(6x - 1)$

D) $\frac{1}{2}(tan^{-1}((3x^2 - x)^2))^{\frac{1}{2}} \bullet \frac{1}{1+(3x^2-x)^4} \bullet 2(3x^2 - x)(6x - 1)$

4. Let $f(x) = (x - 1)^4$. The inverse function of f is denoted by $h(x)$. Find the value of $h'(1)$

A) $\frac{1}{4}$

B) $\frac{1}{16}$

C) 4

D) 16

5. If $x = y^2 - cos(x)$, find $\frac{d^2y}{dx^2}$ at $(0, -1)$.

A) $\frac{3}{4}$

B) $\frac{-1}{4}$

C) $\frac{1}{4}$

D) 1

6. If $f(x) = (x^2 + 1)\sqrt{(x^3 - 2x)}$, find $f'(x)$.

A) $2x\sqrt{x^3 - 2x} + \frac{(x^2+1)(3x^2-2)}{2\sqrt{x^3-2x}}$

B) $2x\sqrt{x^3 - 2x} + \frac{(x^2+1)(3x^2-2)}{\sqrt{x^3-2x}}$

C) $2x\sqrt{x^3 - 2x} - \frac{(x^2+1)(3x^2-2)}{\sqrt{x^3-2x}}$

D) $\sqrt{x^3 - 2x} + \frac{(x^2+1)(3x^2-2)}{\sqrt{x^3-2x}}$

$$f(x) = \sin^{-1}(x + 1)$$

7. The slope of the tangent line to $f(x)$ is equal to 1 at some value of x. What is this value of x?

A) 0

B) $\frac{1}{2}$

C) 1

D) -1

8. Find the slope of the line normal to the graph of $y = x + sin(xy)$ at the point (0,1).

A) 0.5

B) -0.5

C) 2

D) -2

E) 1

9. The position of a particle moving along the x-axis at time t is given by $x(t) = e^{cos(2t)}$, $0 \leq t \leq \pi$. For which of the following values of t will the particle's velocity be equal to 0?

(I) $t = 0$;

(II) $t = \frac{\pi}{2}$;

(III) $t = \pi$;

A) I only

B) II only

C) I and III

D) I and II

E) I, II, and III

10. (CALCULATOR) If $f(x) = x^5\sqrt[3]{2x + 5}$, then $f'(2) \approx$

A) 8.995

B) 171.337

C) 125.874

D) 98.903